Finding Optimal Bitsliced Implementations of $4 \times 4$-bit S-boxes

SKEW 2011
February 17, 2011

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Problem

1. How can we find THE most efficient implementations of s-boxes?
2. Can we find the optimal s-boxes covering all the s-boxes?
Problem

1. How can we find THE most efficient implementations of s-boxes?
2. Can we find the optimal s-boxes covering all the s-boxes?

- S-boxes limited to
  - $4 \times 4$-bit s-boxes
  - Invertible s-boxes
Architecture

- Software implementation using bitslicing
- 4+1 register
- Instruction set
  - AND
  - OR
  - XOR
  - NOT
  - MOV
- No parallelism
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ASM:
MOV r0 r4
AND r0 r1

r0=0x8888
r1=0xC4CC
r2=0xF0F0
r3=0xFF00
r4=0xA4AA

r0=0xA4AA
r1=0xC4CC
r2=0xF0F0
r3=0xFF00
r4=0xA4XX
Search method

- Enumerating all s-boxes in order of cost function
  - No heuristics
- Limited to applications with monotonously increasing cost functions
Affine equivalence:

- Classification according to affine equivalence
- Definition: $S_1(x) = B(S_2(Ax \oplus a) \oplus b)$
- Properties regarding linear and differential cryptanalysis invariant
ASM:
MOV r0 r4
AND r0 r1
r0=0x8888
r1=0xCCCC
r2=0xF0F0
r3=0xFF00
r4=0xA AAA

r0=0xA AAA
r1=0xCCCC
r2=0xF0F0
r3=0xFF00
r4=0xXXXX
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Reducing the branching factor

- Rule set from D. A. Osvik\(^1\)
  - S-box invertible
  - No double negation
  - Reading before overwriting
  - Uninitialised values cannot be read
  - Double nodes are dismissed

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\(^1\) Dag Arne Osvik: Speeding up Serpent. AES Candidate Conference 2000
Advanced caching
Advanced caching
Advanced caching
Advanced caching
Advanced caching

- Initial approach: dismissing nodes that are equal
- New approach: using affine equivalences
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Overview

- Searched until cost of 12 instructions
- more than 2 month on 8 Xeon cores with 64GB RAM
- 272 out of 302 classes found
- Cover 90% of all s-boxes
- For each of these classes:
  - Representative
  - Assembly code
### Linear and differential properties

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</table>

|       | MDP        | $1/8$ | $1/4$ | $3/8$ | $1/2$ | $5/8$ | $3/4$ | $7/8$ | $1$ |      |        |        |        |        |        |        |        |        |        |      |        |        |        |        |        |        |        |      |
|-------|------------|-------|-------|-------|-------|-------|-------|-------|-----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-----|-------|-------|-------|-------|-------|-------|-----|
| min. cost | -         |   9  |  10  |   6  |   9  |   6  |     - |   0  |     |        |        |        |        |        |        |        |        |        |        |      |        |        |        |        |        |        |      |
‘Smallest s-box ever’

- 9 instructions
- MDP = \frac{1}{4}
- MLP = \frac{1}{2} + \frac{1}{4}

**ASM code**

```
0 MOV r4 r0
1 AND r0 r1
2 XOR r0 r2
3 OR r2 r1
4 XOR r2 r3
5 AND r3 r0
6 XOR r3 r4
7 AND r4 r2
8 XOR r1 r4
r0 r1 r2 r3
```
## Compared with literature

<table>
<thead>
<tr>
<th>Cipher</th>
<th>S-box</th>
<th>Class</th>
<th>cost rep.</th>
<th>cost s-box inst. (cycl.)</th>
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<tbody>
<tr>
<td>Serpent</td>
<td>$S_4$, $S_5$</td>
<td>9</td>
<td>11</td>
<td>19 (10)</td>
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<tr>
<td></td>
<td>$S_4^{-1}, S_5^{-1}$</td>
<td>10</td>
<td>12</td>
<td>19 (10)</td>
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<tr>
<td></td>
<td>$S_0^{-1}, S_1$</td>
<td>14</td>
<td>10</td>
<td>18 (10)</td>
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<td>$S_0, S_1^{-1}$</td>
<td>15</td>
<td>10</td>
<td>18 (9)</td>
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<td>$S_2, S_2^{-1}, S_6, S_6^{-1}$</td>
<td>16</td>
<td>11</td>
<td>16 (8)</td>
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<td>$S_3, S_3^{-1}, S_7, S_7^{-1}$</td>
<td>not found</td>
<td>-</td>
<td>18 (10)</td>
</tr>
<tr>
<td>Luffa</td>
<td>$Q$</td>
<td>16</td>
<td>11</td>
<td>16 (6)</td>
</tr>
<tr>
<td>Noekeon</td>
<td>$S = S^{-1}$</td>
<td>13</td>
<td>9</td>
<td>16</td>
</tr>
</tbody>
</table>
A new design approach

Old approach

1. Designing the parts other than s-box
   - specifications get refined more and more
2. Finding s-boxes that fulfil the requirements

New approach

1. Choosing an s-box class
2. Selecting the most efficient representative as s-box
3. Designing the other components of the cipher
Open problems and future research

- Verifying the new design approach
- Affine equivalence and the NOT instruction
- More advanced architectures (SSE, parallelisation)
- Using other classification criteria
Conclusion

- An approach to systematically search efficient implementations of s-boxes has been presented.
- Most s-box classes have been found.
  - Interesting tradeoffs
  - Compared with literature
- New design approach has been proposed.
Questions