Finding Optimal Bitsliced Implementations of 4×4-bit S-boxes

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Problem

1. How can we find THE most efficient implementations of s-boxes?
2. Can we find the optimal s-boxes covering all the s-boxes?

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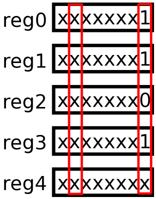
- S-boxes limited to
  - 4×4-bit s-boxes
  - Invertible s-boxes

Architecture

- Software implementation using bitslicing
- 4+1 register
- Instruction set
    - AND
    - OR
    - XOR
    - NOT
    - MOV
- No parallelism



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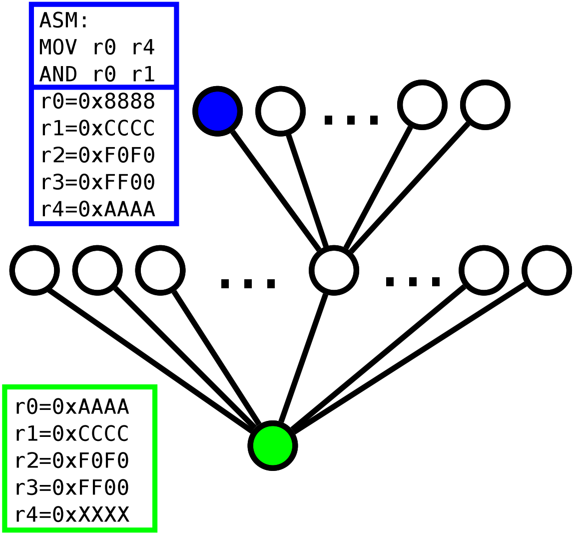
Search

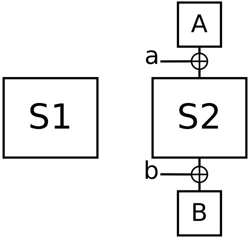

ASM:
MOV r0 r4
AND r0 r1
r0=0x8888
r1=0xCCCC
r2=0xF0F0
r3=0xFF00
r4=0xAAAA
r0=0xAAAA
r1=0xCCCC
r2=0xF0F0
r3=0xFF00
r4=0xXXXX

Search method

- Enumerating all s-boxes in order of cost function
  - No heuristics
- Limited to applications with monotonously increasing cost functions

Equivalence

- Affine equivalence:
    - Classification according to affine equivalence
    - Definition: $S_1(x) = B(S_2(Ax \oplus a) \oplus b)$
    - Properties regarding linear and differential cryptanalysis invariant



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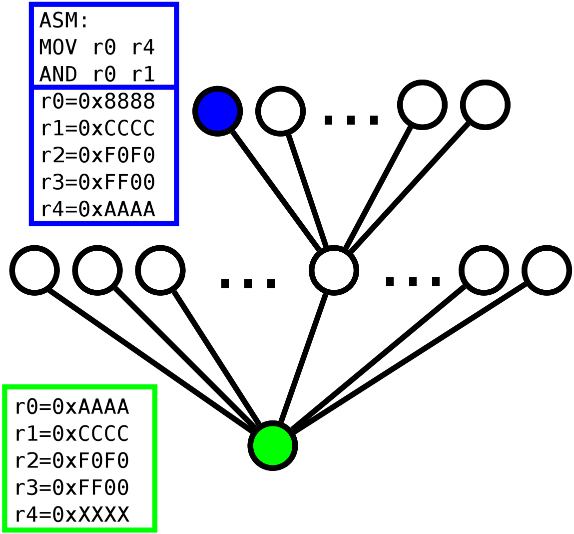
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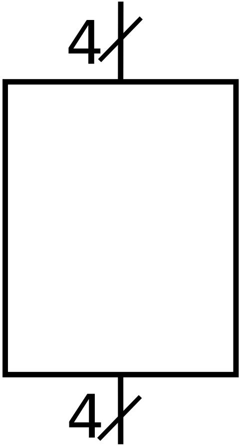
Reducing the branching factor

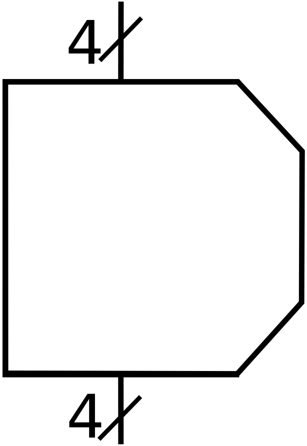
- Rule set from D. A. Osvik[1]
  - S-box invertible
  - No double negation
  - Reading before overwriting
  - Uninitialised values cannot be read
  - Double nodes are dismissed

[1] Dag Arne Osvik: Speeding up Serpent. AES Candidate Conference 2000

Advanced caching



Advanced caching



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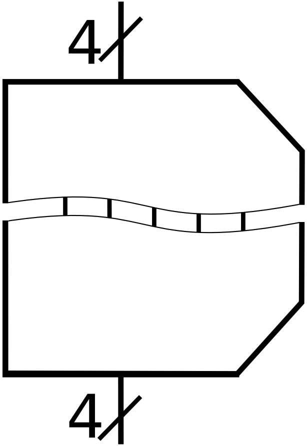

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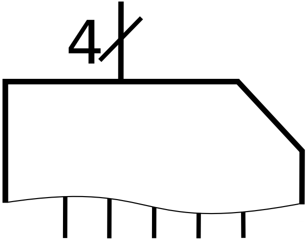
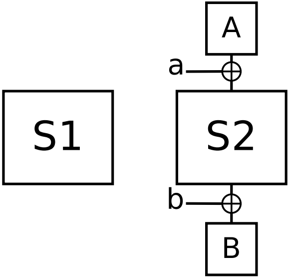
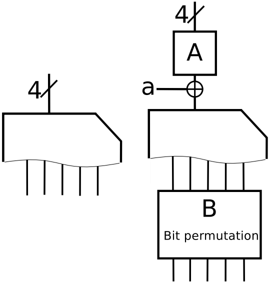


Advanced caching

- Initial approach: dismissing nodes that are equal
- New approach: using affine equivalences





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Overview

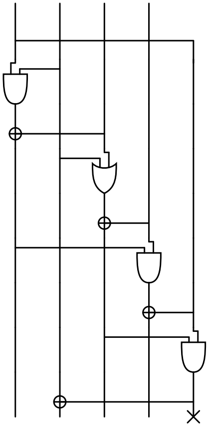
- Searched until cost of 12 instructions
- more than 2 month on 8 Xeon cores with 64GB RAM
- 272 out of 302 classes found
- Cover 90% of all s-boxes
- For each of these classes:
  - Representative
  - Assembly code

Linear and differential properties

MLP $-1/2$	$1/8$	$1/4$	$3/8$	$1/2$
$\lvert c \rvert$	$1/4$	$1/2$	$3/4$	1
min. cost	-	9	9	0

MDP	$1/8$	$1/4$	$3/8$	$1/2$	$5/8$	$3/4$	$7/8$	1
min. cost	-	9	10	6	9	6	-	0

'Smallest s-box ever'



- 9 instructions
- MDP $= 1/4$
- MLP $= 1/2 + 1/4$

ASM code

```
0 MOV r4 r0
1 AND r0 r1
2 XOR r0 r2
3 OR r2 r1
4 XOR r2 r3
5 AND r3 r0
6 XOR r3 r4
7 AND r4 r2
8 XOR r1 r4
r0 r1 r2 r3
```

Compared with literature

Cipher	S-box	Class	cost rep.	cost s-box inst. (cycl.)
Serpent	S_4, S_5	9	11	19 (10)
	S_4^{-1},S_5^{-1}	10	12	19 (10)
	S_0^{-1},S_1	14	10	18 (10)
	S_0,S_1^{-1}	15	10	18 (9)
	S_2,S_2^{-1},S_6,S_6^{-1}	16	11	16 (8)
	S_3,S_3^{-1},S_7,S_7^{-1}	not found	-	18 (10)
Luffa	Q	16	11	16 (6)
Noekeon	$S = S^{-1}$	13	9	16

A new design approach

Old approach

1. Designing the parts other than s-box
    - specifications get refined more and more
2. Finding s-boxes that fulfil the requirements

New approach

1. Choosing an s-box class
2. Selecting the most efficient representative as s-box
3. Designing the other components of the cipher

Contents

Open problems and future research

- Verifying the new design approach
- Affine equivalence and the NOT instruction
- More advanced architectures (SSE, parallelisation)
- Using other classification criteria

Conclusion

- An approach to systematically search efficient implementations of s-boxes has been presented
- Most s-box classes have been found
  - Interesting tradeoffs
  - Compared with literature
- New design approach has been proposed

Questions

Questions?